Smaller Drawdowns, Higher Average and Risk-Adjusted Returns for Equity Portfolios, Using Options and Power-Log Optimization Based on a Behavioral Model of Investor Preferences

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Working Paper

June 2019

The authors thank St. Mary's College of California for research support.

and Financiometrics Inc. for use of their Power-Log Optimizer®.

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Abstract

We use a Power-Log utility optimization algorithm based on a behavioral model of investor preferences, along with either a call, or put option overlay to reverse the negative skewness of monthly S&P 500 index returns, and produce portfolios with smaller drawdowns and far higher risk-adjusted returns than the S&P 500 index. All the optimal portfolios have positively skewed returns, which are preferred by investors. Optimal portfolios containing the call have higher average returns than the S&P500 index, and also much higher average and risk adjusted returns than portfolios containing the put, except for the most conservative portfolios.

Smaller Drawdowns, Higher Average and Risk-Adjusted Returns for Equity Portfolios, Using Options and Power-Log Optimization Based on a Behavioral Model of Investor Preferences

Power-Log optimization is an expected utility maximization method for portfolio construction that is based on a behavioral model of investor preferences, where the utility for losses is modeled differently than the utility for gains (Kahneman and Tversky [1979], Tversky and Kahneman's [1991]). When some assets in a portfolio have skewed returns, Kale (2006) shows that Power-Log optimization produces portfolios that have positively skewed returns and are ideal for controlling downside risk, which are both preferred by investors. Other methods for controlling downside risk typically trade off arbitrary measures of risk against return, and have been described as ad-hoc methods (Leland [1999]). We use Power-Log optimization along with option overlays to take advantage of the extreme positive skewness in returns of call and put options, to reverse the negative skewness of monthly equity portfolio returns and produce portfolios with smaller drawdowns and far higher risk-adjusted returns. The optimal portfolios containing the call option also have far higher average returns than equity portfolios. We use the S&P 500 index to represent equity portfolios, and construct portfolios containing the index, treasuries and either a call option on the index, or a put option on the index. The portfolios containing the call have smaller maximum drawdowns and higher Sortino ratios than the S&P 500 index, except for the riskiest optimal portfolios. The portfolios containing the put also have smaller maximum drawdowns, and higher Sortino ratios than the index, though their overall performance is not as good as that of the portfolios with the call option.

Kale [2006] used simulation with hypothetical data to show that when a portfolio consisting of an equity index, a call option on the index and a riskless asset is optimized with Power-Log utility functions, the risk and return characteristics of the portfolios are superior to that of corresponding portfolios produced with mean-variance optimization (Markowitz [1952]), or by using power utility functions (Grauer and Hakansson [1982]). Kale and Sheth [2016] conducted a backtest of the Power-Log optimization methodology with annual returns over a twenty-year period, using market data for the S&P500 index, an at the money call option on the index and treasury securities to show that the performance of the optimal Power-Log utility portfolios is superior to that of the corresponding mean-variance optimal portfolios. Interestingly, the Kale and Sheth study showed that not only is the downside risk of the optimal Power-Log portfolios substantially lower than that for the corresponding mean-variance portfolios with matching expected returns, but that even the standard deviation of return is lower for the optimal Power-Log portfolios since they perform much better during major market shocks. Kale and Lim [2017] showed the benefits of reversing the negative skewness of value portfolios, which have returns that are even more negatively skewed than growth portfolios. Here we extend that research to all equity portfolios.

The description of Power-Log utility functions and optimal portfolio construction that follows is based on Kale [2006], Kale and Sheth [2016], and Kale and Lim [2017]. Power-Log utility functions start by incorporating the central idea of behavioral economics and prospect theory, that investors view gains and losses asymmetrically (Kahneman and Tversky [1979], Tversky and Kahneman's [1991]), and then superimpose risk aversion across the entire range of returns so that Power-Log utility functions conform to the Friedman-Savage [1948] axioms for risk-averse utility functions. Best and Grauer (2016 and 2017) showed that prospect theory's S-shaped utility function's risk-seeking behavior on the downside results in optimal portfolios "…that are extremely unstable over different decision horizons and with annual data are prone to risk bankruptcy," which, of course, is unacceptable for the vast majority of investors. By replacing prospect theory's risk-seeking behavior on the downside with risk-averse behavior, Power-Log utility functions convert prospect theory's descriptive model of speculation into a normative model for the investment of savings that conforms closely to investor preferences.

A Power-Log utility function is defined as,

$$U = \ln(1+r) \quad \text{for } r \geq 0 \qquad (3)$$

$$= \frac{1}{\gamma}(1+r)^{\gamma} \quad \text{for } r < 0$$

where,

r portfolio return

γ downside power, is less than or equal to 0

This function combines the scalable power utility function for losses with the log utility function for gains. Fig. 1 shows examples of Power-Log utility functions with downside powers -2 and -20 from Kale and Sheth [2016].

The log utility function, which is functionally equivalent to the Kelly criterion (Kelly [1956]), is well known for its portfolio growth-maximization property. The log utility function is a risk averse utility function, and the power utility function is also a risk averse utility function for powers less than or equal to zero. Thus the hybrid Power-Log utility function conforms to the Friedman-Savage [1948] axioms for a risk-averse utility function.

Selecting a downside power of zero for the Power-Log utility function is equivalent to using a log utility function for losses (Grauer and Hakansson [1982]), which makes the entire utility function a log utility function. Portfolios selected with this utility function are typically very risky. To add downside protection, investors can use a negative downside power. For example, changing the downside power from 0 to -2 increases the penalty for losses, and will produce portfolios with smaller downside risk, while the utility associated with gains remains unchanged. Changing the downside power from -2 to -20 increases the penalty for losses substantially, and results in quite conservative portfolios. The continuous differentiability of the Power-Log utility function also allows us to use very fast optimization algorithms for portfolio selection. The algorithm used for this study is an accelerated conjugate direction method for nonlinear mathematical programming developed by Best and Ritter (1976) that is particularly well suited for this type of optimization problem[1].

To construct optimal portfolios, the expected utility criterion developed by Von Neumann and Morgenstern [1944], and Savage [1964] gives us the following optimization problem.

Select assets weights, w_i, to:

Maximize

$$E(U) = \sum_s p_s U_s \qquad (4)$$

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[1] The algorithm implementation used in this study is the Power-Log Optimizer® from Financiometrics Inc.

where,

s scenario s, and the summation is over all scenarios

p_s probability of scenario s

U_s utility of portfolio return r_s in scenario s, where utility is defined by the Power-Log function in Equation 3

The portfolio return, r_s, is calculated as an investment weighted average of the returns to the assets in the portfolio,

$$r_s = \sum_i w_i r_{is} \tag{5}$$

where,

i asset i, and the summation is over all assets in the portfolio

w_i investment weight of asset i in the portfolio

r_{is} return to asset i in scenario s

The portfolio optimization algorithm uses the entire joint distribution of asset returns. All the moments and cross-moments of asset returns, including mean, variance, skewness, kurtosis, correlation, coskewness, cokurtosis, and more are taken into account. As a result, if some of the assets have return distributions that are skewed, the optimization process produces portfolios with positively skewed return distributions.

To demonstrate the methodology and compare the use of a call option with the use of a put option in the portfolio, we use a one-month horizon for portfolio construction, and market data at 1:05 pm Eastern Standard Time, USA, on January 17, 2018 from the Schwab website. Using middle of the day option price quotes avoids the turmoil in bid and ask quotes at the close of trading. On January 17 we forecast the one-month joint distribution of returns for the S&P 500 total returns index, an at the money put on the S&P 500 index, or an at the money call on the index, and a one-month treasury security. This joint return distribution is then used in the optimization for portfolio construction.

On January 17, 2018, the one-month treasury's yield was listed as 1.31% on the federal government's website www.treasury.gov, which converts to 0.1092% per month. To forecast the distribution for the S&P 500 total returns index, we start by calculating the historical total monthly returns for ^SP500TR with data from Yahoo!Finance from February 1988 through December 2017, the 359 observations of monthly data available at the time. For the forecast, we kept the average return at its historical average value of 0.9242%, but adjusted the standard deviation of historical returns by using information about future volatility from the VIX (volatility index). The VIX index was 11.31% at 1:05 pm Eastern Standard Time on January 17, 2018, which is equivalent to a monthly volatility of 3.2649%. Since the historical standard deviation of the S&P500 total returns index is very slightly higher than that of the S&P500 price index, we made a minor adjustment to the standard deviation of the VIX to estimate the standard deviation of the S&P500 total returns index.

To forecast the monthly return distribution of the call option on the S&P 500 index, we start by forecasting the monthly return distribution for the S&P 500 price index using data for ^GSPC from Yahoo!Finance from February 1988 through December 2017, a monthly volatility of 3.2649%, and the same procedure as for the S&P 500 total returns index above. At 1:05 pm

Eastern Standard Time on January 17, 2018, the S&P 500 index value was 2,797.55. The closest to the money call with an expiration date of February 16, had one month to expiration, a strike price of 2,800, a bid price of $26.9, an ask price of $27.6, and therefore a midpoint call price of $27.25. For brevity we will refer to this call option as an at-the-money, or ATM call. Using the S&P 500 index value of 2,797.55 and its monthly return distribution forecast, we forecast the values for the index one month later, and then calculate the corresponding expiration values for the call, given the strike price of 2,800. With a call price of $27.25 and the forecast of expiration values, we forecast the monthly return distribution for the call option.

The constant return for the one-month treasury, the distribution for the S&P 500 total returns index and the distribution for the ATM call option on the index, together constitute the forecast of the joint return distribution for the three assets that is needed for constructing portfolios with the Power-Log Optimizer. Table 1 shows sections of the monthly joint return distribution forecast for the treasury, the S&P 500 total returns index and the call option.

To forecast the return distribution of the closest to the money put option we use the same procedure as for the call option. The put has the same expiration date and strike price as the call, and for brevity we will refer to it as the ATM put. At 1:05 pm Eastern Standard Time on January 17, 2018, the put had a bid price of $27.6, and an ask price of $28.3, with a midpoint put price of $27.95. Table 2 shows sections of the monthly joint return distribution forecast for the treasury, the S&P 500 total returns index and the ATM put.

We construct optimal portfolios containing the treasury, the S&P 500 total returns index and the ATM call, and compare their risk and return characteristics to those of the S&P 500 total returns index. Next we construct portfolios containing the treasury, the S&P 500 total returns index and the ATM put, and compare their risk and return characteristics to those of the S&P 500 index as well as those of the optimal portfolios containing the ATM call.

Optimal Portfolios

We construct optimal portfolios using Power-Log utility functions with downside powers that range from 0 to -100. The different downside powers represent different levels of investor aversion to losses. A downside power of 0 corresponds to the log utility function for the entire range of positive and negative returns and produces the riskiest portfolios. Downside powers around -9 correspond to the aversion to losses for average investors, and more negative downside powers result in more conservative portfolios. To make it easier to interpret the results, we impose a "no short sales" constraint on all the assets.

Table 3 shows the optimal investment weights for the optimal portfolio with the call option for downside powers from -100 to 0. The downside power -100 produces the most conservative portfolio, while the downside power 0, which corresponds to a log utility function, produces the most aggressive portfolio. For downside power -100, the investment in the treasury security is 98.97%, which makes this an extremely conservative portfolio with a tiny investment of 1.03% in the call option. Clearly the call option has a role to play even in the most conservative portfolio in this set. As the downside power increases from -100 to 0, the investment in the treasury drops to 72.50%, while the investment in the call option rises to 27.50%, while the investment in the S&P 500 index remains at zero throughout. The combination of a treasury investment and a call option completely dominates investment in the S&P 500 index for conservative, average and aggressive investors. Fig. 2 shows the optimal investment weights for all 1,001 optimal portfolios with call options that we constructed for downside powers increasing from -100 to 0 in steps of 0.1. The riskiest portfolios on the right side of the chart where downside power is close to zero, have a substantial position in the call option, which is likely to be too high for most investors, but portfolios constructed with more negative downside powers have risk and return characteristics that are attractive for virtually all investors, as shown in the figures that follow.

To get a sense of the call option's notional exposure, consider an optimal $100 million portfolio constructed with downside power -9, which reflects the aversion to losses for an average investor. For this portfolio the investment in treasury bills, S&P 500 index and the call option are 92.76%, 0% and 7.24% respectively, as shown in Table 3. This portfolio will have $92.76 million invested in treasury bills, and $7.24 million in call options and nothing in the S&P 500 index. Given the call option price of $27.25, the number of calls in the portfolio is 7,240,000 / 27.25, or 265,688.07. Given the S&P 500 index price of $2,797.55, the call position's notional exposure is 265,688.07 × $2,797.55, or $743,275,660.23, which implies a leverage of 7.43 to 1. This large notional exposure might appear scary, but this entire notional exposure is on the upside, which is a very good thing since it has the potential for generating very large gains. This leverage does not exist on the downside, since the call option's payoff is zero on the downside, and in addition the downside is covered by the huge position in treasury bills, which constitute 92.76% of the portfolio.

Table 4 shows the optimal investment weights for the optimal portfolio with the put option for downside powers from -100 to 0. For downside power -100, the investment weights are 0% in the treasury, 98.86% in the S&P 500 index and 1.14% in the put option. The 1.14% put exposure tempers the 98.86% equity exposure enough to make this portfolio a low risk portfolio. This portfolio is very different from the optimal portfolio for downside power -100, when the call option is used, which has a zero investment in the S&P 500. As the downside power increases from -100, for the optimal portfolios with the put option the investment in the S&P 500 index keeps increasing and eventually gets to 99.67%, while the put investment decreases to 0.33%. Interestingly, even for the most aggressive optimal portfolio, Power-Log optimization builds in the benefit of some level of downside protection, which of course promotes higher growth over time.

For the put option's notional exposure, once again consider a $100 million optimal portfolio constructed with downside power -9. Based on the investment weights in Table 4, this portfolio will have $98.98 million invested in the S&P 500, $1.02 million in put options, and nothing in treasury bills. Given the put price of $27.95, the number of puts is 1,020,000 / 27.95, or 36,493.74, which implies a notional exposure of 36,493.74 × $2,797.55, or $102,093,062.34. The resulting 1.02 to 1 leverage is devoted entirely to protecting the downside, since the put option has no upside potential.

Fig. 3 shows the maximum monthly drawdown for the optimal portfolios and the S&P 500 index. It is based on the forecast of the monthly joint return distribution, as are the results shown in all the charts that follow. The S&P500 index has a maximum drawdown of 13.37%. This forecast value reflects the disastrous performance of the S&P500 index in October 2008. For all the optimal portfolios containing the put, the maximum drawdown is smaller than that for the S&P500 index. For the optimal portfolios containing the call, the maximum drawdown is also smaller than that for the S&P500 index, except for the riskiest portfolios constructed with downside powers greater than -3. Both sets of optimal portfolios can deliver far smaller maximum drawdowns than the S&P500 index.

Fig. 4 shows the average monthly return for a given maximum monthly drawdown. In this chart and the ones that follow, the maximum monthly drawdown is the measure of risk, specifically downside tail risk, and it is on the X-axis. The S&P500 index has an average monthly return of 0.92%, and a maximum monthly drawdown of 13.37%. For the same level of drawdown tail risk as the S&P 500 index, the corresponding optimal portfolio with the call option delivers an average return that is about 8% per month higher. While the extreme drawdown in the S&P500 index is a rare event, this chart emphasizes how much better off the investor will be, when that tail risk is controlled by investing in the optimal portfolios containing the call option. Looking at tail risk in terms of Value at Risk, VaR, at the 95% confidence level, VaR for the S&P500 index is 4.93%, which implies that 5% of the time, the S&P500 index is likely to have a monthly drawdown of 4.93%, or higher. The optimal portfolio with the call option constructed with a downside power of -14.9 has a maximum monthly drawdown of 4.93%, which is smaller than 5% of the likely S&P500 index drawdowns. This optimal portfolio also has an average monthly return of 3.42%, which is more than three times higher than the monthly average return for the S&P500 index. Only the most conservative optimal portfolios, those with maximum drawdowns smaller than 1.13%, have average returns that are smaller than that for the S&P 500 index. In contrast, the average returns for all the optimal portfolios containing the put option are slightly lower than the average return for the S&P500, which implies that paying for preventing losses is not sufficient for outperforming the S&P500 index. In Fig. 4, the chart for the optimal portfolios with the put is shorter than those with the call, since the largest maximum monthly drawdown for optimal portfolios containing the put is 9.14%.

Fig. 5 shows the skewness of return for a given maximum monthly drawdown. While the skewness of the S&P 500 return is -0.60, all the optimal portfolios with the call options have a positive skewness of 1.33, which is of course the skewness of the call option return, since these portfolios contain only the call option and a riskless investment in treasury bills. Positive skewness in returns is well known as a desirable characteristic of portfolio returns, thus this reversal in skewness from substantially negative for the S&P 500 index, to substantially positive for the optimal portfolios containing the call option is very significant. The skewness for the most conservative portfolios containing the put is about the same as that for the most conservative portfolios containing the call, but as the maximum drawdown increases, the skewness for these portfolios drops steadily to almost zero, which is still better than that for the S&P 500 index.

In addition to drawdown, another popular measure of downside risk is negative semideviation below zero (Markowitz, 1959), which we call ***loss deviation***. Standard deviation treats the extreme positive returns of positively skewed non-normal distributions as major contributors to risk, and is inappropriate as a measure of risk for portfolios with positively skewed return distributions. Loss deviation is based on losses only, and conforms to investor association of risk with losses, not gains. In addition, investors evaluate gains and losses differently as captured in the investor preferences represented in the Power-Log utility functions, which makes loss deviation a far more appropriate measure of risk.

We define loss deviation as,

$$Loss\ Deviation = \sqrt{\sum_{s} p_s (r_s^-)^2} \tag{7}$$

where,

r_s^- portfolio return in scenario s if it is negative, zero otherwise, and the sum is over all scenarios.

p_s probability of scenario s.

With loss deviation as our measure risk we calculate the Sortino ratio,

$$Sortino\ Ratio = \frac{Average\ Return}{Loss\ Deviation} \tag{8}$$

The Sortino ratio comes in many flavors, and this definition corresponds to the total return approach to portfolio management, where the portfolio is not managed relative to a benchmark, such as a riskfree asset, or a normal portfolio. This type of total return approach was also emphasized by Myron Scholes (2017), in his talk at the University of San Francisco titled, "A (R)evolution in Asset Management." Of course, if desired the optimization can be set up to construct portfolios that are managed relative to a benchmark. Fig. 6 shows the Sortino ratio for a given maximum monthly drawdown. The Sortino ratio for the S&P 500 index is 0.45, and the Sortino ratio for all the optimal portfolios with the call option is substantially higher across the board. This is also true for all the optimal portfolios containing the put option, which have better Sortino ratios than the portfolios containing the call option for the most conservative portfolios. Besides positive skewness in returns, optimal Power-Log portfolios also deliver much better risk adjusted returns.

Scenario Analysis

While backtesting remains popular in the investment field, it can often lead to conclusions that do not hold up in the future. Sharpe describes the problems with such tests in Sharpe and Litterman (2014): "… I've been around so long that I've seen spectacular empirical results that don't stand up after somebody does the same analysis in a different country, for a different time period, or after waiting a year or two to see what happens." Scenario analysis is a useful alternative to get a feel for what could happen to portfolio performance given different states of the market. In place of a backtest, we provide a forward looking scenario analysis. Using our forecasts for the S&P500 and the optimal portfolio return distributions, we look at different possible states for the S&P500 monthly return.

Figure 7 shows the monthly return forecasts for the optimal portfolio with the call option constructed with a downside power of -9, for the full range of forecasts of the S&P 500 return. For S&P 500 losses greater than 7.1%, the optimal portfolio has smaller losses than the S&P 500, and for S&P 500 returns greater than 1.2% the optimal portfolio has substantially higher gains. For S&P500 returns between -7.1% and 1.2%, the optimal portfolio's performance is not as good as that for the S&P 500. Thus, the optimal portfolio does not give us a "free lunch," where the portfolio's return is better than that for the S&P 500 for every scenario, but the optimal portfolio does provide downside protection when the S&P 500 is faring poorly, along with substantially higher returns when the index does well, which produces far higher average return for the optimal portfolio as shown in Fig. 4. This return profile is significantly different from that for a protective put, or portfolio insurance, which has an upside that is lower than that for the S&P 500, because of the cost of the put option. The downside power of -9 corresponds to the aversion to losses for an average investor. For investors with greater aversion to losses, modeled with more negative downside powers, downside protection improves but upside potential is lower, though still higher than that for the S&P 500.

Figure 8 shows the monthly return forecasts for the optimal portfolio with the put option constructed with a downside power of -9, for the full range of forecasts of the S&P 500 return. The performance of the optimal portfolio is better than that for the S&P 500 when S&P 500 return is lower than -0.8%, but there is no upside benefit, and the overall return profile for the optimal portfolio is similar to that for portfolio insurance. Of course, the benefit of using Power-Log optimization instead of portfolio insurance, is that with Power-Log optimization the risk and return characteristics of the optimal portfolios correspond directly to investor preferences for gains and losses, instead of the arbitrary loss limit that is used in portfolio insurance. For investors with greater aversion to losses, modeled with downside powers more negative than -9, downside protection improves with optimal portfolios containing the put option. If the investor's goal is to control losses without regard to gains, then the optimal portfolio containing the put does a better job than the optimal portfolio containing the call, but if the investor's goal is to generate higher returns with lower risk, then the Sortino ratios in Figure 6 demonstrate the clear superiority of the optimal portfolios with the call for all but the most conservative investors.

Conclusion

Power-Log optimization is an expected utility maximization method that is based on a behavioral model of investor preferences, where the utility for losses is modeled differently than the utility for gains. When some assets in a portfolio have skewed returns, Power-Log optimization produces portfolios that have positively skewed returns, which are preferred by investors, and are ideal for controlling downside risk. We use Power-Log optimization along with option overlays to take advantage of the extreme positive skewness in returns of call and put options, to reverse the negative skewness of equity portfolio returns and produce portfolios with smaller drawdowns and far higher average and risk-adjusted returns. We use the S&P 500 index to represent equity portfolios, and construct portfolios containing the index, treasuries and either a call option on the index, or a put option on the index.

The optimal portfolios containing the call option provide downside protection and substantial upside potential, which is well above the upside potential of the S&P 500 index. They have smaller maximum drawdowns and higher Sortino ratios than the S&P 500, except for the very riskiest portfolios. The portfolios containing the put option provide downside protection, but on the upside their performance lags the performance of the S&P 500 index by a small amount. These portfolios also have also have smaller maximum drawdowns, and higher Sortino ratios than the S&P 500, though their overall performance is not as good as that of the portfolios containing the call option.

Power-Log optimization with option overlays adds to the traditional methods of portfolio construction. It can deliver better downside protection along with far higher average and risk-adjusted returns than pure equity portfolios.

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Table 1

Monthly Joint Return Distribution Forecast with Call Option (%)

No.	Treasury	S&P500	ATM Call
1	0.11	3.94	251.87
2	0.11	-2.31	-100.00
3	0.11	1.07	-16.32
4	0.11	0.88	-68.05
5	0.11	3.88	263.78
⋮	⋮	⋮	⋮
355	0.11	0.43	-89.82
356	0.11	1.84	65.47
357	0.11	2.06	89.36
358	0.11	2.65	138.16
359	0.11	1.08	-12.95

Table 2

Monthly Joint Return Distribution Forecast with Put Option (%)

No.	Treasury	S&P500	ATM Put
1	0.11	3.94	-100.00
2	0.11	-2.31	163.63
3	0.11	1.07	-100.00
4	0.11	0.88	-100.00
5	0.11	3.88	-100.00
⋮	⋮	⋮	⋮
355	0.11	0.43	-100.00
356	0.11	1.84	-100.00
357	0.11	2.06	-100.00
358	0.11	2.65	-100.00
359	0.11	1.08	-100.00

Table 3

Optimal Portfolios Containing the Call Option

Downside	Investment Weights (%)		
Power	Treasury	S&P500	ATM Call
-100	98.97	0.00	1.03
-50	98.13	0.00	1.87
-25	96.65	0.00	3.35
-20	95.99	0.00	4.01
-15	94.99	0.00	5.01
-10	93.27	0.00	6.73
-9	92.76	0.00	7.24
-8	92.17	0.00	7.83
-7	91.46	0.00	8.54
-6	90.60	0.00	9.40
-5	89.53	0.00	10.47
-4	88.17	0.00	11.83
-3	86.36	0.00	13.64
-2	83.79	0.00	16.21
-1	79.82	0.00	20.18
0	72.50	0.00	27.50

Table 4

Optimal Portfolios Containing the Put Option

Downside	Investment Weights (%)		
Power	Treasury	S&P500	ATM Put
-100	0.00	98.86	1.14
-50	0.00	98.88	1.12
-25	0.00	98.91	1.09
-20	0.00	98.92	1.08
-15	0.00	98.94	1.06
-9	0.00	98.98	1.02
-5	0.00	99.03	0.97
0	0.00	99.67	0.33

Fig. 1

Power-Log Utility Function with Downside Power -2 and -20

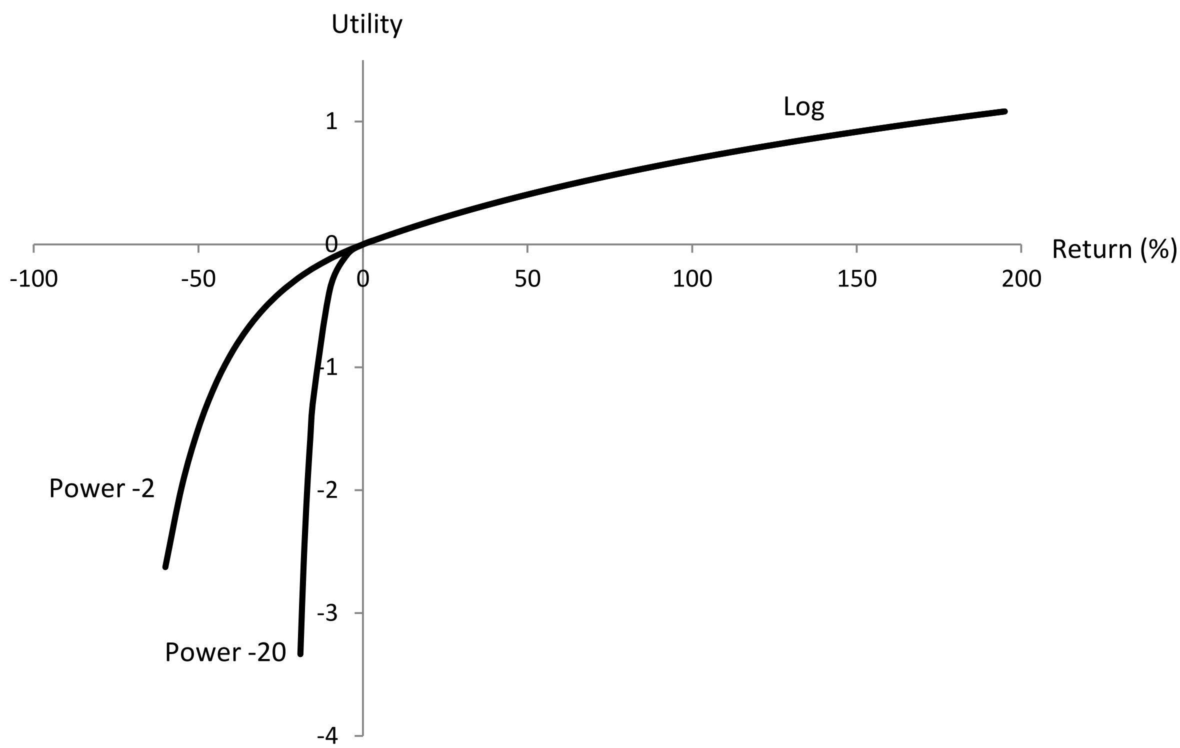

Fig. 2

Optimal Portfolio Weights with the Call Option

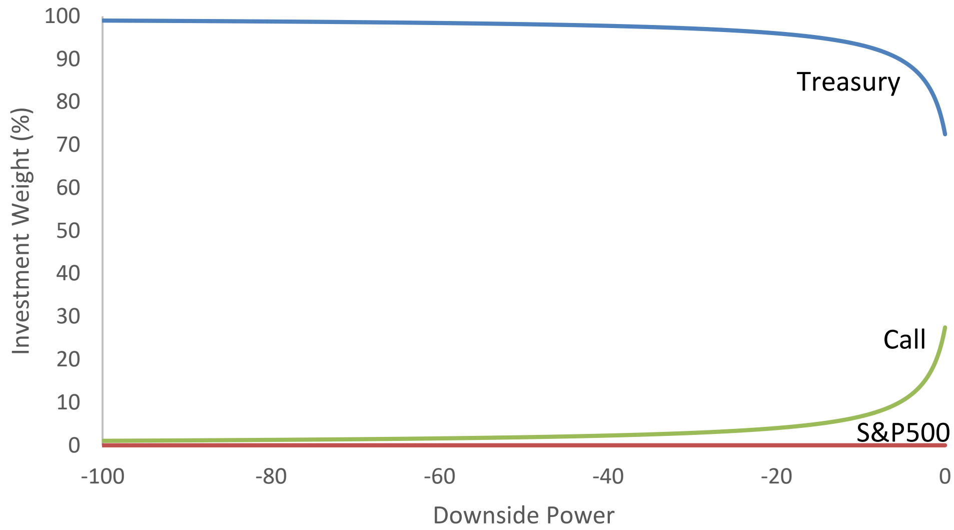

Fig. 3

Maximum Monthly Drawdown for Optimal Portfolios

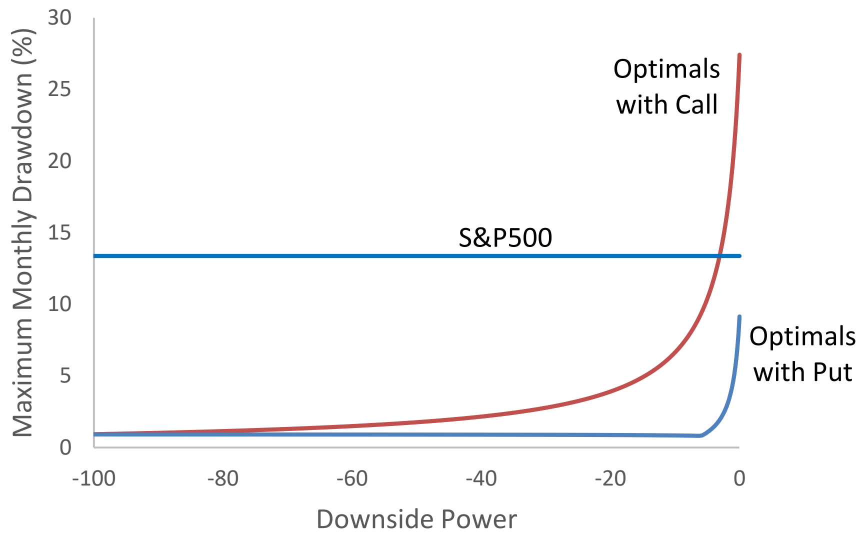

Fig. 4

Average Monthly Return for the Optimal Portfolios

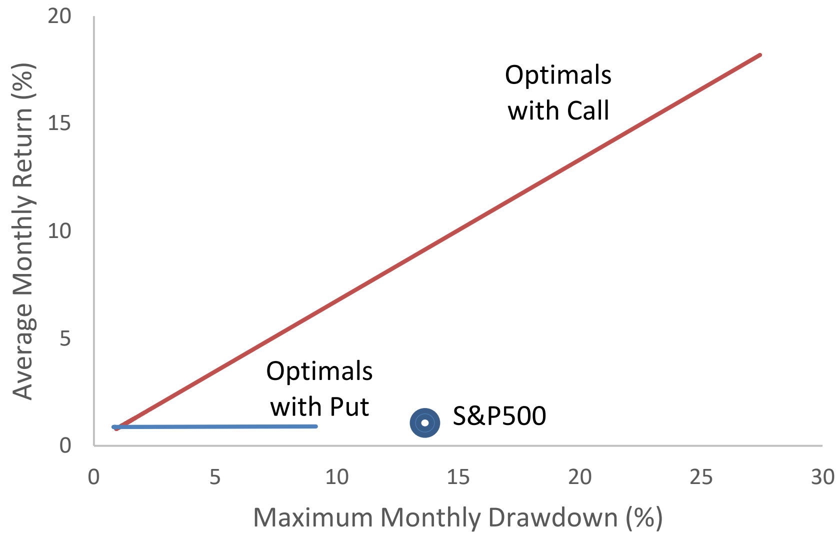

Fig. 5

Skewness for the Optimal Portfolios

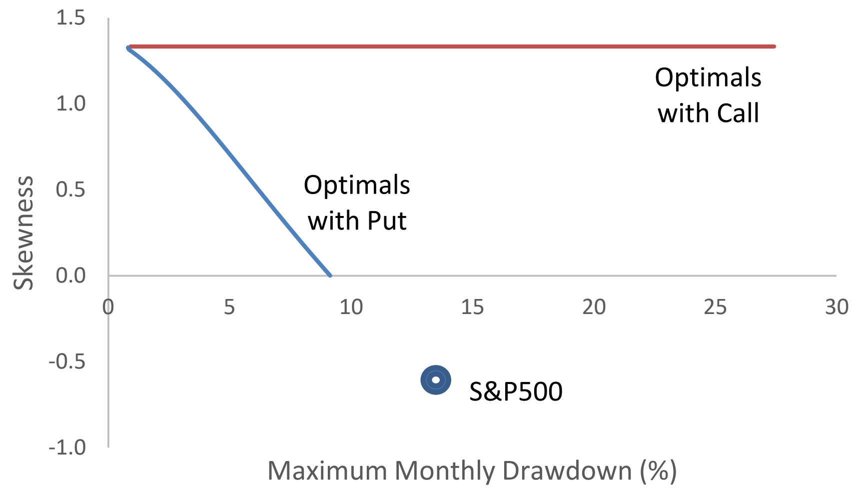

Fig. 6

Sortino Ratios for the Optimal Portfolios

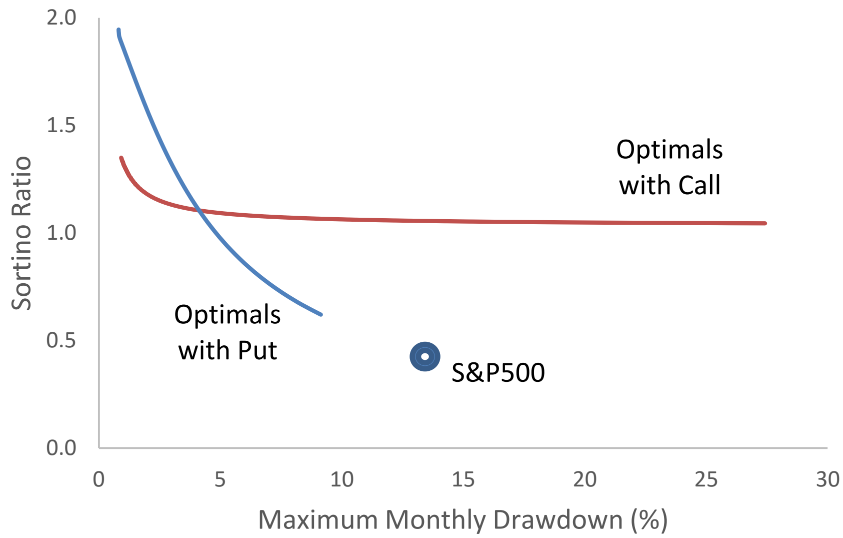

Fig. 7

Optimal Portfolio Return Scenarios with the Call Option

Downside Power -9

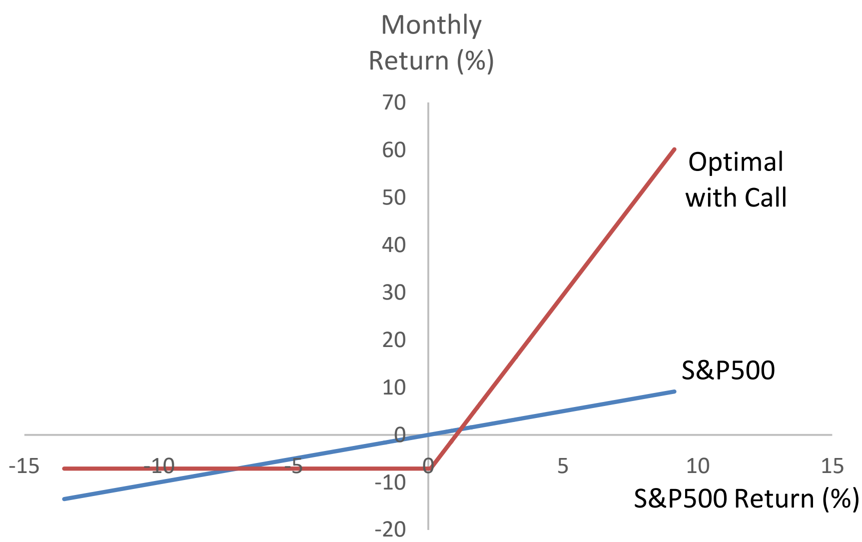

Fig. 8

Optimal Portfolio Return Scenarios with the Put Option

Downside Power -9